Solutions to JEE Main - 4 | JEE - 2024

PHYSICS

SECTION-1

1.(B) $g - T = a$ (vertical block)

$T - \mu g = a$ (horizontal block)

Add, $(1-\mu)g = 2a \Rightarrow 0.8g = 2a \Rightarrow a = 0.4g$

2.(B) Work, $W = \vec{F} \cdot \vec{s} = \left(3\hat{i} - 4\hat{j}\right) \cdot \left(\hat{i} - \hat{j}\right) = 7J$

3.(A) To reach at point D. Block has to make complete vertical circle BCD.

In the limiting case V_B has to be $\sqrt{5gR}$ for looping the loop (condition for complete circle)

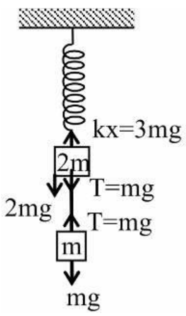
Now applying energy conservation at A & B

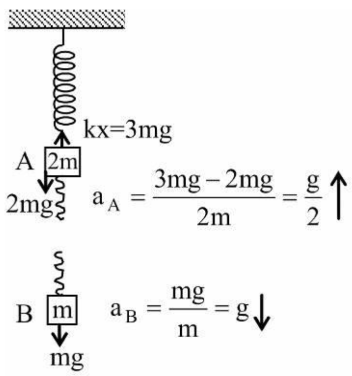
$$mgh = \frac{1}{2}mv_B^2 \Rightarrow h = \frac{5}{2}R$$

If $h \geq \frac{5}{2}R$, then $v_B \geq \sqrt{5gR}$ & block can easily reach to D. But if $h < \frac{5}{2}R$, then $v_B < \sqrt{5gR}$ block can not reach to D.

Required condition $\Rightarrow h \geq \frac{5}{2}R$

4.(B)

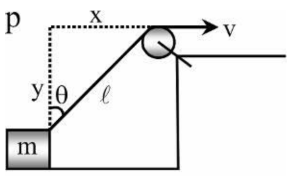




5.(B) $x^2 + y^2 = \ell^2$

$2x\frac{dx}{dt} + 0 = 2\ell\frac{d\ell}{dt}$ [y = constant]

or $\left|\frac{dx}{dt}\right| = \frac{\left|\frac{d\ell}{dt}\right|}{(x/\ell)} = \frac{v}{\sin\theta}$



6.(C) $\frac{mv^2}{R} = N\cos\theta$

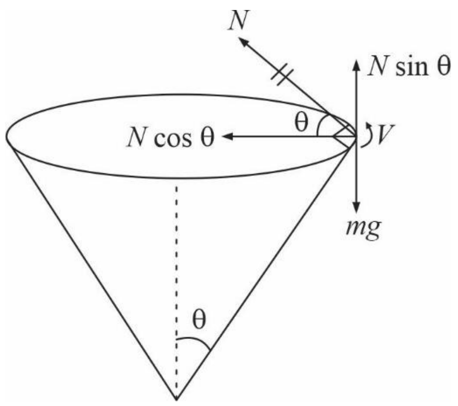
$mg = N\sin\theta$

Divide $\cot\theta = \frac{v^2}{Rg}$

Also, $\cot\theta = \frac{h}{R}$

$\therefore \quad v^2 = gh$

$v = \sqrt{gh} = \sqrt{9.8 \times 0.2}\, m/s \Rightarrow v = 1.4\, m/s$



7.(A) $E_i + W_{\text{friction}} = E_f$

$\Rightarrow \quad \dfrac{Mv_0^2}{2} - \mu Mg\,x = +\dfrac{1}{2}kx^2$

$\Rightarrow \quad \dfrac{v_0^2}{2} - (0.9)(10)(0.4) = 50(0.4)^2 \quad \Rightarrow \quad v_0^2 = 23.2$

$\therefore \quad v_0 = \sqrt{23.2}\,m/s = 4.8\,m/s$

8.(D) Let the length of the string be L and the velocity of the particle when it is at the lowest point be v

Conserving energy, $\quad mgL(1-\cos\theta) = \dfrac{1}{2}mv^2 \quad \Rightarrow \quad v^2 = 2gL(1-\cos\theta)$

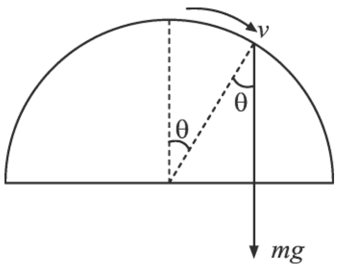
Therefore, acceleration at the lowest point,

$$a = \frac{v^2}{L} = 2g(1-\cos\theta) = 2g\left(1-\frac{\sqrt{3}}{2}\right) = \left(2-\sqrt{3}\right)g$$

9.(D) $W_{\text{air}} + W_{\text{gravity}} = \Delta k$

$\Rightarrow \quad W_{\text{air}} + 0 = \dfrac{m}{2}\dfrac{v_0^2}{4} - \dfrac{m}{2}v_0^2 \quad \therefore \quad W_{\text{air}} = -\dfrac{3mv_0^2}{8}$

10.(C) At the time of leaving contact $N = 0$



$\therefore \quad mg\cos\theta = \dfrac{mv^2}{R} = \dfrac{m(2gh)}{R}$

$\therefore \quad \cos\theta = \dfrac{2h}{R} = \dfrac{2\left[\dfrac{R}{4} + R(1-\cos\theta)\right]}{R}$

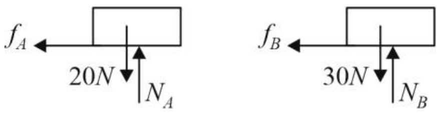
On solving the equation, we get $\cos\theta = 5/6$; $\quad \theta = \cos^{-1}\left(\dfrac{5}{6}\right)$

11.(C) F – 2T = 6a and T = 4 × 2a

$\therefore \quad$ F – 16 a = 6a

$\Rightarrow \quad$ a = $\dfrac{F}{22}$ $\Rightarrow$ a = 1m/s^2

12.(C) Let the speed of turntable be ω.



From FBD of blocks we get

$(f_A)_{\max} = \mu_A N_A = (0.4)(20) = 8N$

and $\quad (f_B)_{\max} = \mu_B N_B = (0.9)(30) = 27N$

Also, $\quad (f_A)_{\max} = m_A\omega^2 r_A$

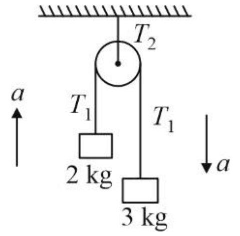
$8 = (2)(\omega^2)(0.25) \quad \Rightarrow \quad \omega = 4$ rad/sec

So, $\quad f_B = m_B\omega^2 r_B$

$f_B = (3)(4)^2(0.5) = 24N$

(Hence, $f_B < (f_B)_{\max}$)

13.(D) From the diagram,



$$T_1 - 2g = 2a$$

$$3g - T_1 = 3a$$

Solving, we get

$$a = 2 \text{ m/s}^2, \; T_1 = 24N$$

We know that the net force on the pulley must be zero because it is massless

Therefore, $T_2 = 2T_1 = 48N$

14.(B) Net force on the particle,

$$\vec{F} = \frac{d}{dt}(\vec{p}(t)) = -A\omega\sin(\omega t)\hat{i} + B\omega\cos(\omega t)\hat{j}$$

So, at $t = \frac{2\pi}{3\omega}$, $\vec{F} = -A\omega\sin\left(\frac{2\pi}{3}\right)\hat{i} + B\omega\cos\left(\frac{2\pi}{3}\right)\hat{j} = -\frac{\sqrt{3}}{2}A\omega\hat{i} - \frac{1}{2}B\omega\hat{j}$

15.(C) This can be understood in two ways:

(i) Instantaneous power due to a force $\vec{F}$ is $P = \vec{F}\cdot\vec{v}$

Here, $\vec{v}$ is the instantaneous velocity

Since, $\vec{F}$ is the force of gravity and hence vertically downward, the dot-product becomes:

$$P = F_y v_y$$

Let us take the upward direction to be positive

So, in the upward journey, $v_y > 0$, and hence $P < 0$ (Since $F_y < 0$ always)

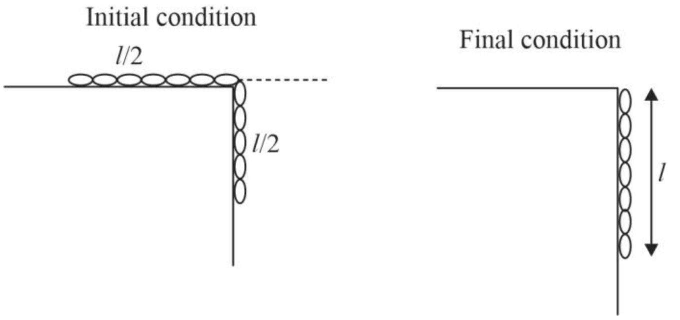
And in the downward journey, $v_y < 0$ and hence $P > 0$.

(ii) Since gravity is the only force acting, the power delivered by gravity must be equal to the rate of change of kinetic energy of the particle

So, as in the upward journey, speed is decreasing, power is negative

And in the downward journey, speed is increasing, so power is positive.

16.(C)



Since no friction anywhere & normal forces does no work. We can apply energy conservation between initial & final condition.

Assuming zero gravitational PE at table level.

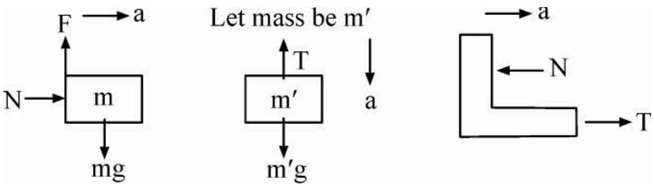
$$(PE)_i = (m_{hanging})g\,(y_{cm})_{hanging} \qquad = \frac{-m}{2}g\left(\frac{l}{4}\right) = -\frac{mgl}{8}$$

Similarly $(PE)_f = -mg\left(\frac{l}{2}\right) = -\frac{mgl}{2}$

$$(KE)_i + (PE)_i = (KE)_f + (PE)_f$$

$$0 - \frac{mgl}{8} = \frac{1}{2}mv^2 - \frac{mgl}{2} \qquad \Rightarrow \quad v = \sqrt{\frac{3}{4}gl}$$

17.(D)



$N = ma$ …(i)

$F = mg$ …(ii)

$m'g - T = m'a$ …(iii)

$T - N = Ma$ …(iv)

Solving (i) and (ii)

$\mu N = mg$ or $\mu ma = mg \quad \Rightarrow \quad a = \dfrac{g}{\mu}$

Adding (i), (iii) and (iv)

$m'g = (m + m' + M)\dfrac{g}{\mu} \quad \Rightarrow \quad m'g - \dfrac{m'g}{\mu} = (m + M)\dfrac{g}{\mu}$

$m'(\mu - 1) = (m + M) \quad \Rightarrow \quad m' = \dfrac{m + M}{(\mu - 1)}$

18.(D) $W = \int_i^f \vec{F} \cdot \vec{ds} = \int_{(1, 2)}^{(2, 3)} (3x^2\hat{i} + 2y\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$

$= \int_1^2 3x^2 dx + \int_2^3 2y\, dy$

$= [x^3]_1^2 + [y^2]_2^3 = (8 - 1) + (9 - 4) = 12J$

19.(B) $F_x = -\dfrac{dU}{dx} = -10x + 20$

$F_x = 0 \quad \Rightarrow \quad x = 2m$ is equilibrium point

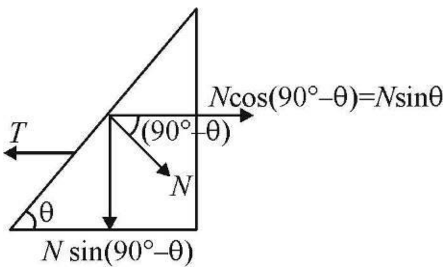
i.e. $a = 0$

$\Rightarrow$ Velocity is max. at $x = 2m$

As $x = 2m$ is stable equilibrium point

20.(C) $T = N\sin\theta$ and $N = mg\cos\theta$

$T = mg\cos\theta\sin\theta = \dfrac{mg}{2}\sin 2\theta$



SECTION – 2

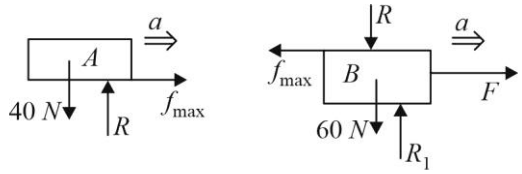
1.(5) Both will show same tension $\therefore$ Reading of is = 5 N.

2.(2) $P = \dfrac{3t^2}{2} \Rightarrow \vec{F}.\vec{v} = \dfrac{3t^2}{2}$

$$m\left(\frac{dv}{dt}\right)v = \frac{3t^2}{2} \Rightarrow m\int_0^v v\,dv = \frac{3}{2}\int_0^v t^2 dt$$

$$2\frac{v^2}{2} = \frac{3}{2}\left|\frac{t^3}{3}\right|_0^2 \Rightarrow v^2 = 4 \text{ or } v = 2 \text{ m s}^{-1}$$

3.(4) From FBD of the blocks we get



$$R = 40N \text{ and } f_{max} = \mu R = 24N$$

And for block A, $f_{max} = m_A a$

$$\Rightarrow \quad a = \frac{24}{4} = 6 \text{ m/s}^2$$

Also, for block B, $F - f_{max} = m_B a$

$$\Rightarrow \quad F - 24 = 36$$

$$\Rightarrow \quad F = 60N = 15 \times 4N \qquad \text{So,} \quad n = 4$$

4.(1) Let the velocity of the block after it has moved down by a distance x be v

Conserving energy,

Loss in gravitational PE = Gain in kinetic energy + Gain in elastic PE

$$\Rightarrow \quad mgx = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

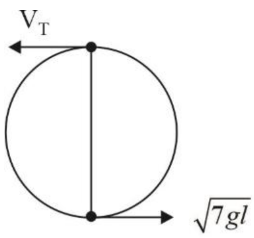
$$\Rightarrow \quad v^2 = \frac{2mgx - kx^2}{m}$$

We are given that $m = 1$ kg $\quad k = 100$ N/m and $\quad x = 0.1$ m

Therefore, $v^2 = \dfrac{2(1)(10)(0.1) - (100)(0.1)^2}{1} = 1$

$$\Rightarrow \quad v = 1 \text{ m/s}$$

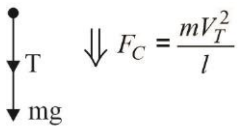
5.(2) Minimum T will be at top point. From energy conservation



$$\frac{1}{2}mV_T^2 + mg(2l) = \frac{1}{2}m(7gl)$$

$$\Rightarrow \quad V_T = \sqrt{3gl}$$

At top point FBD,



(T + mg) provide centripetal force mV_T^2 / l

$$\Rightarrow \quad T + mg = \frac{mV_T^2}{l} \qquad \Rightarrow \quad T = 2\text{mg}$$

CHEMISTRY

SECTION-1

1.(A) Shimla has lowest atmospheric pressure hence liquid will boil first in this city. Boiling of a liquid takes place when the vapour pressure becomes equal to the atmospheric pressure.

2.(A) For curve B, value of PV is constant and for an ideal gas plot of PV vs P is a straight line.

3.(D) If 3/8th of the air is expelled out then remaining air $= \frac{5}{8}$

In the open vessel volume and pressure are both constant but moles are decreasing.

$$\therefore \quad n_1T_1 = n_2T_2 \quad \therefore \quad n_1 \times 300 = \left(\frac{5}{8}n_1\right)T_2$$

$$\therefore \quad T_2 = \frac{8}{5} \times T_1 = \frac{8}{5} \times 300 = 480\,K = 480 - 273 = 207°C$$

4.(D) As per Boyles law

PV = constant

$\log P = -\log V + \log(\text{constant})$

5.(C) $$\frac{r_{O_2}}{r_{unknown}} = \frac{\frac{v_1}{t_1}}{\frac{v_2}{t_2}} = \sqrt{\frac{M_{unknown}}{M_{O_2}}}$$

$$\Rightarrow \quad \frac{100 \times 150}{100 \times 75} = \sqrt{\frac{M_{unknown}}{32}}$$

$$\Rightarrow \quad M_{unknown} = 128 \quad \therefore \quad V.D. = \frac{128}{2} = 64$$

6.(A) Average speed $= \sqrt{\frac{8RT}{\pi M}}$

Both N_2 and CO have same molecular mass.

So, for same average speed, temperature should be same

$\therefore \quad T = 200\,K = -73°C$

7.(B) **(A)** Maximum density of CO_2 in gaseous phase will be at point 'A'.

Now $\quad d = \frac{PM}{RT} = \frac{60 \times 44}{R \times 300} = 107\,gm/lt = 0.107\,gm/cc$

Or $\quad d = \frac{\text{mass of gas}}{\text{Volume of gas}} = \frac{44\,gm}{440\,cc} = 0.1$

(B) Density of liquid CO_2 at 60 atm (at point B)

At point B only liquid CO_2 is present, so we find out the density of liquid CO_2 only at point B.

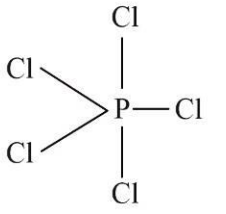
$d = \frac{M}{V} = \frac{44}{0.044} = 1\,gm/cc$

(C) BD represent liquid state.

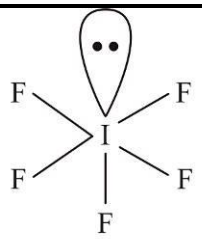
(D) Z can be greater than 1 at 27°C.

For example at point 'A'. Z is 1.07, So wrong.

8.(C) **(I)**



sp^{3}d hybridised
trigonal bipyramidal



sp^{3}d^2
square pyramidal

(II) Bond angle of $BF_3 = BCl_3 = 120°$

(III) Two carbons are sp-hybridized while one is sp^3 hybridized.

9.(B) $SF_4\ (\mu \neq 0)$ $\quad SiF_4\ (\mu = 0)$

10.(C) π - character is due to $3d\pi \leftarrow 2p\pi$ back bonding 3d orbital of Si and 2p orbital of N.

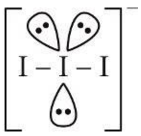
11.(D) Bond order of $He_2^+ \rightarrow 0.5$

$O_2^- \rightarrow 1.5$

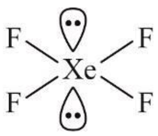
$C_2 \rightarrow 2$

$NO \rightarrow 2.5$

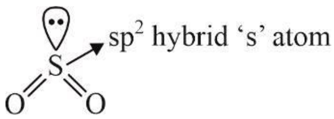
12.(A)

$I_3^- \longrightarrow$ 

$CO_2 \rightarrow O = C = O$

$XeF_4 \longrightarrow$ 

XeF_5^- : Pentagonal planar

13.(B) SO_2 has $p\pi - p\pi$ and $p\pi - d\pi$ bond 

while XeO_3, SO_4^{2-} have only $p\pi - d\pi$ bond. C_2H_4 has only $p\pi - p\pi$ bond.

14.(C) Assuming that no 2s-2p mixing takes place.

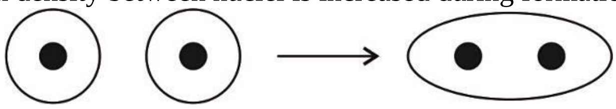
(i) $Be_2 \rightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2$ (diamagnetic)

(ii) $B_2 \rightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \begin{matrix}\pi 2p_x^0 \\ \pi 2p_y^0\end{matrix}$ (diamagnetic)

(iii) $C_2 \rightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \begin{matrix}\pi 2p_x^1 \\ \pi 2p_y^1\end{matrix}, \begin{matrix}\pi^* 2p_x^0 \\ \pi^* 2p_y^0\end{matrix}, \sigma^* 2p_z^0$ (paramagnetic)

(iv) $N_2 \rightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \begin{matrix}\pi 2p_x^2 \\ \pi 2p_y^2\end{matrix}, \begin{matrix}\pi^* 2p_x^0 \\ \pi^* 2p_y^0\end{matrix}, \sigma^* 2p_z^0$ (diamagnetic)

15.(D) Electron density between nuclei is increased during formation of BMO in H_2.



BMO is $\psi_A + \psi_B$ (Linear combination of atomic orbitals) provides constructive interference.

While by destructive interference antibonding MO $\psi_A - \psi_B$ formed.

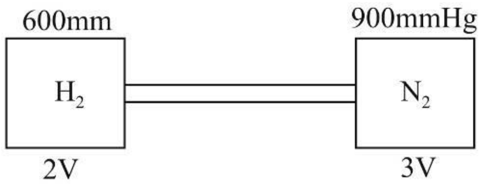
16.(B)

17.(A) Refer NCERT

18.(D) The bond dissociation energy of hydrogen molecule has been found to be (approx.) 436 kJ mol^{-1}.

19.(B) Boiling point of H_2O is more than boiling point of HF. Energy of hydrogen bond varies between 10 to 100 kJ mol^{-1}.

20.(A)



$$3H_2(g) + N_2(g) \longrightarrow 2NH_3(g)$$

$$n_{H_2} = \frac{600 \times 2V}{RT}; \quad n_{N_2} = \frac{900 \times 3V}{RT}$$

Clearly, H_2 is the limiting agent.

$\Rightarrow$ N_2 left $= \frac{2700\,V}{RT} - \frac{400\,V}{RT} = \frac{2300\,V}{RT}$ and NH_3 formed $= \frac{800\,V}{RT}$

$$\Rightarrow \quad P_{new} = \frac{\left(\frac{3100\,V}{RT}\right)RT}{5V} = 620\,mm$$

SECTION – 2

1.(8) Let the number of moles of dihydrogen and dioxygen be 1 and 4.

Mole fraction of $O_2 = \frac{4}{5}$

Partial pressure of dioxygen = Mole fraction $\times$ Total pressure

$$= \frac{4}{5} \times 1 = 0.8 = 8 \times 10^{-1} atm$$

2.(4) At high pressure Z > 1 i.e., repulsive forces are relatively dominant, so gas is less compressible at high pressure, $a \cong 0$.

3.(10) Charge of electron $= 1.6 \times 10^{-19} C$

Dipole moment of $HBr = 1.6 \times 10^{-30} C \times m$

Inter-atomic space (distance) $= 1 Å \quad = 1 \times 10^{-10} m$

Percentage of ionic character in HBr

$$= \frac{\text{Dipole moment of HBr} \times 100}{\text{inter-atomic distance} \times q} = \frac{1.6 \times 10^{-30}}{1.6 \times 10^{-19} \times 10^{-10}} \times 100$$

$$= 10^{-30} \times 10^{29} \times 100 = 10^{-1} \times 100 = 0.1 \times 100 = 10\%$$

4.(2) O_2 and CsO_2 are paramagnetic.

5.(5) SO_3, SF_6, CS_2 are molecules with symmetrical shapes and have zero dipole moment.

MATHEMATICS

SECTION-1

1.(C) $\sum a_i = 10 \times \frac{(2+3)}{2} = 25$; $\sum \frac{1}{h_i} = 10 \times \frac{\left(\frac{1}{2}+\frac{1}{3}\right)}{2} = \frac{25}{6}$

$(g_1 g_2 \ldots\ldots g_{10}) = (2 \times 3)^5 \Rightarrow (g_1 g_2 \ldots\ldots g_{10})^{1/5} = 6 \Rightarrow$ The required product is $= 25 \times \frac{25}{6} \times 6 = 625$

2.(C) $T_r = \frac{8r}{4r^4+1} = \frac{8r}{(4r^4+4r^2+1)-4r^2} = \frac{8r}{(2r^2+1)^2-(2r)^2}$

$$= \frac{8r}{(2r^2-2r+1)(2r^2+2r+1)} = 2\left[\frac{1}{(2r^2-2r+1)} - \frac{1}{(2r^2+2r+1)}\right]$$

$S_n = 2\left[1-\frac{1}{2n^2+2n+1}\right] \Rightarrow S_\infty = 2$

3.(D) By the properties of *AP* and *GP*

$$a_1 + a_{2n} = a_2 + a_{2n-1} = \ldots\ldots = a_n + a_{n+1} = a+b$$

and $g_1 g_{2n} = g_2 g_{2n-1} = \ldots\ldots = g_n g_{n+1} = ab$

$\therefore$ $\frac{a_1+a_{2n}}{g_1 g_{2n}} + \frac{a_2+a_{2n-1}}{g_2 g_{2n-1}} + \ldots\ldots + \frac{a_n+a_{n+1}}{g_2 g_{n+1}} = \frac{a+b}{ab} + \frac{a+b}{ab} + \ldots\ldots + \frac{a+b}{ab} = \frac{n(a+b)}{(ab)} = \frac{2n}{h}$

4.(C) Given that, $\frac{a_1+a_2+\ldots\ldots+a_p}{a_1+a_2+\ldots\ldots+a_q} = \frac{p^2}{q^2}$

$\therefore$ $\frac{\frac{p}{2}[2a_1+(p-1)d]}{\frac{q}{2}[2a_1+(q-1)d]} = \frac{p^2}{q^2}$

where *d* be a common difference of an AP.

$\Rightarrow$ $\frac{(2a_1-d)+pd}{(2a_1-d)+qd} = \frac{p}{q} \Rightarrow (2a_1-d)(p-q) = 0 \Rightarrow a_1 = \frac{d}{2}$

Now, $\frac{a_6}{a_{21}} = \frac{a_1+5d}{a_1+20d} = \frac{\frac{d}{2}+5d}{\frac{d}{2}+20d} = \frac{11}{41}$

5.(B) The general term is $T_n = \frac{\frac{n}{2}\cdot\frac{n+1}{2}}{1^3+2^3+3^3+\ldots\ldots+n^3} = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$

$\therefore$ $S_n = 1 - \frac{1}{n+1} = \frac{n}{n+1}$

6.(A) Let the two quantities be a and b. Then a, A_1, A_2, b are in AP.

$\therefore$ $A_1 - a = b - A_2 \Rightarrow A_1 + A_2 = a+b$ **. . . .(i)**

Again, a, G_1, G_2, b are in GP.

$\therefore$ $\frac{G_1}{a} = \frac{b}{G_2} \Rightarrow G_1 G_2 = ab$ **. . . .(ii)**

Also, a, H_1, H_2, b are in HP.

$\therefore \quad \frac{1}{H_1}-\frac{1}{a}=\frac{1}{b}-\frac{1}{H_2} \quad \Rightarrow \quad \frac{1}{H_1}+\frac{1}{H_2}=\frac{1}{a}+\frac{1}{b}$

$\Rightarrow \quad \frac{H_1+H_2}{H_1H_2}=\frac{a+b}{ab}=\frac{A_1+A_2}{G_1G_2}$ [From equation (i) and (ii)]

$\Rightarrow \quad \frac{G_1G_2}{H_1H_2}=\frac{A_1+A_2}{H_1+H_2}$

7.(A) Since, $a_1, a_2, a_3, ..., a_n$ from an AP.

$\therefore \quad a_2-a_1=a_4-a_3=......=a_{2n}-a_{2n-1}=d$

Let $S=a_1^2-a_2^2+a_3^2-a_4^2+......+a_{2n-1}^2-a_{2n}^2$

$=(a_1-a_2)(a_1+a_2)+(a_3-a_4)(a_3+a_4)+......+(a_{2n-1}-a_{2n})(a_{2n-1}+a_{2n})$

$=-d(a_1+a_2+......+a_{2n})=-d\left(\frac{2n}{2}(a_1+a_{2n})\right)$ **. . . .(i)**

Also, we know $a_{2n}=a_1+(2n-1)d$

$\Rightarrow \quad d=\frac{a_{2n}-a_1}{2n-1} \quad \Rightarrow \quad -d=\frac{a_1-a_{2n}}{2n-1}$

On putting the value of *d* in equation (i), we get :

$S=\frac{n(a_1-a_{2n})(a_1+a_{2n})}{2n-1}=\frac{n}{2n-1}(a_1^2-a_{2n}^2)$

8.(A) $sin\left[(\omega^{10}+\omega^{23})\pi-\frac{\pi}{4}\right]=sin\left[(\omega+\omega^2)\pi-\frac{\pi}{4}\right]=sin\left(-\pi-\frac{\pi}{4}\right)=-sin\left(\pi+\frac{\pi}{4}\right)=sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$

9.(C) $P=(1+z)(1+z^2)(1+z^4).....\left(1+z^{2^n}\right)=\frac{(1-z)(1+z)(1+z^2)(1+z^4)....\left(1+z^{2^n}\right)}{(1-z)}=\frac{\left(1-z^{2^{n+1}}\right)}{(1-z)}$

10.(C) Given $x+\frac{1}{x}=1 \Rightarrow x^2-x+1=0 \Rightarrow x=\frac{1\pm\sqrt{3}i}{2} \Rightarrow x=\frac{1+\sqrt{3}i}{2}, x=\frac{1-\sqrt{3}i}{2} \Rightarrow x=-\omega^2$ or $x=-\omega$.

$\therefore \quad x^{20}+x^{30}+x^{40}=\omega+1+\omega^2=0$

11.(B) $\left(\frac{2cos^2\phi/_2+i\,2\,sin\phi/_2 . cos\phi/_2}{2cos^2\phi/_2-i\,2\,sin\,\phi/_2\,.\,cos\,\phi/_2}\right)^n$

$=\left(\frac{2cos\,\phi/_2+i\;sin\phi/_2}{cos\phi/_2-i\;sin\,\phi/_2}\times\frac{cos\,\phi/_2+i\,sin\,\phi/_2}{cos\,\phi/_2+i\,sin\phi/_2}\right)^n=\frac{(cos\,\phi+i\;sin\,\phi)^n}{1}=cos\,n\phi+i\;sin\,n\phi$

12.(A) $\because \quad x_r=\cos\left(\frac{\pi}{2^r}\right)+i\sin\left(\frac{\pi}{2^r}\right)$

$\therefore \quad x_1=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$

$x_2=\cos\frac{\pi}{2^2}+i\sin\frac{\pi}{2^2}$

… … … …

… … … …

$\therefore \quad x_1x_2x_3....=\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)\left(\cos\frac{\pi}{2^2}+i\sin\frac{\pi}{2^2}\right)....$

$$= \cos\left(\frac{\pi}{2}+\frac{\pi}{2^2}+....\right)+i\sin\left(\frac{\pi}{2}+\frac{\pi}{2^2}+....\right)$$

$$= \cos\left(\frac{\frac{\pi}{2}}{1-\frac{1}{2}}\right)+i\sin\left(\frac{\frac{\pi}{2}}{1-\frac{1}{2}}\right) = \cos\pi + i\sin\pi = -1$$

13.(D) $\dfrac{(cos\theta - i\,sin\theta)^4}{(sin\theta + i\,cos\theta)^5} = \dfrac{1}{i^5}\dfrac{(cos\theta - i\,sin\theta)^4}{(cos\theta - i\,sin\theta)^5} = \dfrac{1}{i}(cos\theta + i\,sin\theta) = sin\theta - i\,cos\theta$

14.(B) Given, $\alpha = e^{i\frac{2\pi}{n}}$, i.e., α is nth roots of unity i.e. of equation $x^n - 1 = 0$

Then, $x^n - 1 = (x-1)(x-\alpha)(x-\alpha^2)\ldots(x-\alpha^{n-1})$

$\Rightarrow \quad \dfrac{x^n-1}{x-1} = (x-\alpha)(x-\alpha^2)\ldots(x-\alpha^{n-1})$

Put $x = 11$, then

$$\frac{11^n - 1}{10} = (11-\alpha)(11-\alpha^2)\ldots(11-\alpha^{n-1})$$

15.(B) Given, $z = \left(\dfrac{\sqrt{3}}{2}+\dfrac{i}{2}\right)^5 + \left(\dfrac{\sqrt{3}}{2}-\dfrac{i}{2}\right)^5$

$$= \left(cos\frac{\pi}{6} + i\,sin\frac{\pi}{6}\right)^5 + \left(cos\frac{\pi}{6} - i\,sin\frac{\pi}{6}\right)^5$$

$$= 2cos\frac{5\pi}{6} = -\sqrt{3} + i0$$

$\therefore \quad Im(z) = 0$

16.(A) $e^{e^{i\theta}} = e^{\cos\theta\,+\,i\sin\theta} = e^{cos\,\theta}\left[cos(sin\theta) + i\,sin(sin\theta)\right]$

So, $\quad Re\left(e^{e^{i\theta}}\right) = e^{cos\,\theta}\left[cos(sin\theta)\right]$

17.(D) We have, $\quad |zw| = 1$

$\therefore \quad arg(z) - arg(w) = \dfrac{\pi}{2}$

$\Rightarrow \quad arg\left(\dfrac{z}{w}\right) = \dfrac{\pi}{2} \qquad \Rightarrow \quad \dfrac{z}{w} = \left|\dfrac{z}{w}\right| \times e^{i\pi/2} = i\dfrac{|z|}{|w|}$

$\Rightarrow \quad z = iw\dfrac{|z|}{|w|} \qquad$ **. . . .(i)**

Now, $\quad \bar{z}w = -i\bar{w}\dfrac{|z|}{|w|}\cdot w \qquad \left[\text{As } \bar{z} = -i\bar{w}\dfrac{|z|}{|w|}, \text{ by equation (i)}\right]$

$$= -i\frac{|w|^2|z|}{|w|} = -i|w||z|$$

$\therefore \quad \bar{z}w = -i$

18.(A) Given that, z_1, z_2 and z_3, z_4 are two pairs of complex conjugate.

$\therefore \quad z_2 = \overline{z}_1$ and $z_4 = \overline{z}_3$

$\Rightarrow \quad z_1 \cdot z_2 = z_1 \cdot \overline{z}_1 = |z_1|^2 \qquad \left[\because z\overline{z} = |z|^2\right] \qquad$ **....(i)**

$z_3 \cdot z_4 = z_3 \cdot \overline{z}_3 = |z_3|^2 \qquad \left[\because \overline{z} = |z|^2\right] \qquad$ **....(ii)**

Now, $arg\left(\frac{z_1}{z_4}\right) + arg\left(\frac{z_2}{z_3}\right) = arg\left(\frac{z_1 z_2}{z_4 z_3}\right) \qquad \left[\because arg(z_1) + arg(z_2) = arg(z_1 \cdot z_2)\right]$

$= arg\left(\frac{|z_1|^2}{|z_3|^2}\right)$ [Using equations (i) and (ii)]

$= arg\left(\frac{|z_1|^2}{|z_3|^2}\right) = 0 \qquad \left[\because arg|z|^2 = 0\right]$

19.(C) Given, $|z_1| = |z_2| = \ldots = |z_n| = 1 \quad \Rightarrow \quad \frac{1}{z_1} = \overline{z}_1, \frac{1}{z_2} = \overline{z}_2$........

Now, $\left|\frac{z_1 + z_2 + z_3 + \ldots + z_n}{\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \ldots + \frac{1}{z_n}}\right|$

$= \left|\frac{z_1 + z_2 + z_3 + \ldots + z_n}{\overline{z}_1 + \overline{z}_2 + \overline{z}_3 + \ldots + \overline{z}_n}\right|$

$\left[\because \overline{z}_1 + \overline{z}_2 + \overline{z}_3 + \ldots + \overline{z}_n = \overline{z_1 + z_2 + z_3 + \ldots + z_n}\right]$

$= \left|\frac{z_1 + z_2 + z_3 + \ldots + z_n}{\overline{z_1 + z_2 + z_3 + \ldots + z_n}}\right| \qquad \left[\because \left|\frac{z_1}{z_2}\right| = \left|\frac{z_1}{z_2}\right|\right]$

$= \left|\frac{z_1 + z_2 + z_3 + \ldots + z_n}{z_1 + z_2 + z_3 + \ldots + z_n}\right| \qquad \left[\because |\overline{z}| = |z|\right]$

$= 1$

20.(A) Given system of equations are :

$Re\left(z^2\right) = 0$ and $|z| = 2$

Let $\quad z = x + iy$

$\therefore \quad z^2 = x^2 - y^2 + 2ixy$

Also, $\quad |z| = 2 \Rightarrow \sqrt{x^2 + y^2} = 2$

$\Rightarrow \quad x^2 + y^2 = 4$

Hence, $\quad Re\left(z^2\right) = 0$

$\Rightarrow \quad x^2 - y^2 = 0 \qquad$ **....(i)**

$|z| = 2$

$\Rightarrow \quad x^2 + y^2 = 4 \qquad$ **....(ii)**

Solving (i) and (ii), we get

X	2	2	–2	–2
y	2	–2	–2	2

SECTION – 2

1.(2) Given $x^4 - 4x^3 + ax^2 + bx + 1 = 0$

Let x_1, x_2, x_3, x_4 be the 4 positive roots

$x_1 + x_2 + x_3 + x_4$

A.M. = $\dfrac{x_1 + x_2 + x_3 + x_4}{4} = 1$

G.M. = $(x_1 \cdot x_2 \cdot x_3 \cdot x_4)^{1/4} = 1$

Hence, A.M. = G.M. $\Rightarrow x_1 = x_2 = x_3 = x_4 = 1 \Rightarrow a = 6,\ b = -4$

2.(8) The two A.P.'s will be 10, 17, 24, ... and 150, 137, 124 . . . . .

$\Rightarrow \quad 10 + (n-1)7 = 150 + (n-1)(-13)$

$\Rightarrow \quad n = 8$

3.(6) Put $z = x + iy$, we get $Re\left(\dfrac{z-3}{z+i}\right) = \dfrac{2}{5}$

$\Rightarrow \quad \dfrac{x(x-3) + y(y+1)}{x^2 + (y+1)^2} = \dfrac{2}{5} \Rightarrow \quad 5\left(x^2 + y^2 - 3x + y\right) = 2\left(x^2 + y^2 + 2y + 1\right)$

$\Rightarrow \quad 3x^2 + 3y^2 - 15x + y - 2 = 0$ or $x^2 + y^2 - 5x + \dfrac{1}{3}y - \dfrac{2}{3} = 0$

Hence, on comparing, we get : $a = -5,\ b = \dfrac{1}{3},\ c = \dfrac{-2}{3}$

$\Rightarrow \quad (b - c - a) = 6$

4.(–4) Let $Z = a + ib,\ b \neq 0$ where $Im\ Z = b$

$Z^5 = (a+ib)^5 \quad = a^5 + 5a^4(ib) + 10a^3(ib)^2 + 10a^2(ib)^3 + 5a(ib)^4 + (ib)^5$

$Im Z^5 = 5a^4b - 10a^2b^3 + b^5$

Let $y = \dfrac{Im Z^5}{[Im(Z)]^5} = 5\left(\dfrac{a}{b}\right)^4 - 10\left(\dfrac{a}{b}\right)^2 + 1$

Let $\left(\dfrac{a}{b}\right)^2 = x$ (say), $x \in R^+$

$y = 5x^2 - 10x + 1$

$= 5\left[x^2 - 2x\right] + 1 = 5\left[(x-1)^2\right] - 4$

Hence, $y_{min} = -4$

5.(18) Let x be a real root. Equation real and imaginary part

$x^3 - 6x^2 + 5x + 2a^2 = 0$ . . . .**(i)**

and $2x^3 - 2x^2 - 4x = 0$ . . . .**(ii)**

$2x(x-2)(x+1) = 0$

The given $x = 0, 2$ or -1

If $x = 0 \qquad a = 0$

$x = -1 \Rightarrow a^2 = 6 \Rightarrow a = \pm\sqrt{6}$

$x = 2 \Rightarrow a^2 = 3 \Rightarrow a = \pm\sqrt{3}$

$a \in \left\{0, \sqrt{6}, -\sqrt{6}, \sqrt{3}, -\sqrt{3}\right\}$

$\sum a^2 = 0 + 6 + 6 + 3 + 3 = 18$